Math 115

Spring 2018

Lecture 30

$$? \; a^2 + b^2 = c^2 \; ?$$
$$y = mx + b \quad ? \quad d = rt$$

Class Qz:

① Simplify: $\dfrac{x - \frac{36}{x}}{x - 10 + \frac{24}{x}} = \dfrac{x^2 - 36}{x^2 - 10x + 24} = \dfrac{(x+6)\cancel{(x-6)}}{\cancel{(x-6)}(x-4)}$

LCD = x

$= \boxed{\dfrac{x+6}{x-4}}$

② Solve $\dfrac{2}{x-3} - \dfrac{1}{x+5} = \dfrac{16}{x^2 + 2x - 15}$

LCD = $(x-3)(x+5)$ E.V.: 3 & −5

$2(x+5) - 1(x-3) = 16$

$2x + 10 - x + 3 = 16$

$x + 13 = 16$

$\cancel{x = 3}$

Do not say $\cancel{\{\emptyset\}}$

No Solution

$\{ \quad \} \qquad \emptyset$

The sum of a number and 3 times its reciprocal is $\frac{28}{5}$.

find all such numbers.

Let x be the number,

$$x + 3 \cdot \frac{1}{x} = \frac{28}{5}$$

$$x + \frac{3}{x} = \frac{28}{5}$$

LCD = $5x$

$a = 5 \quad b = -28 \quad C = 15$

$$b^2 - 4ac = (-28)^2 - 4(5)(15)$$

$$= 784 - 300$$

$$= 484$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{28 + 22}{10} = \frac{50}{10} = \boxed{5}$$

$$5x \cdot x + 5x \cdot \frac{3}{x} = 5x \cdot \frac{28}{5}$$

$$5x^2 + 15 = 28x$$

$$5x^2 + 15 - 28x = 0$$

$$5x^2 - 28x + 15 = 0$$

$$ax^2 + bx + C = 0$$

$$= \frac{-(-28) \pm \sqrt{484}}{2(5)} = \frac{28 \pm 22}{10}$$

$$x = \frac{28 - 22}{10} = \frac{6}{10} = \frac{3}{5}$$

Numbers are 5 & 3/5

The difference of an integer and 3 times its reciprocal is $\frac{13}{4}$.

find all such integers.

Let x be some integer,

$$x - 3 \cdot \frac{1}{x} = \frac{13}{4}$$

$$x - \frac{3}{x} = \frac{13}{4}$$

LCD = $4x$

$$4x^2 - 12 - 13x = 0$$

$$4x^2 - 13x - 12 = 0$$

$a = 4 \quad b = -13 \quad C = -12$

$$4x \cdot x - 4x \cdot \frac{3}{x} = 4x \cdot \frac{13}{4}$$

$$4x^2 - 12 = 13x$$

$$b^2 - 4ac = (-13)^2 - 4(4)(-12)$$

$$= 361$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-13) \pm \sqrt{361}}{2(4)} = \frac{13 \pm 19}{8}$$

$$x = \frac{13 + 19}{8} = \frac{32}{8} = \boxed{4}$$

the integer is 4.

$$x = \frac{13 - 19}{8} = \frac{6}{8} = \frac{3}{4}$$

Not an integer

The sum of the reciprocal of two consecutive integers is $\frac{9}{20}$.
Find all such consecutive integers.

Let x & $x+1$ be two cons. integers,

Their reciprocal is $\frac{1}{x}$ & $\frac{1}{x+1}$

$$\boxed{\frac{1}{x}} + \boxed{\frac{1}{x+1}} = \boxed{\frac{9}{20}}$$

LCD = $x(x+1)\cdot 20$ E.V. $0, -1$

$$20(x+1) + 20x = 9x(x+1)$$

$$20x + 20 + 20x = 9x^2 + 9x$$

$$40x + 20 = 9x^2 + 9x$$

$$9x^2 + 9x - 40x - 20 = 0$$

$$9x^2 - 31x - 20 = 0$$

$a = 9$ $b = -31$ $c = -20$

$a = 9$ $b = -31$ $c = -20$

$$b^2 - 4ac = (-31)^2 - 4(9)(-20) = 1681$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-31) \pm \sqrt{1681}}{2(9)}$$

$$= \frac{31 \pm 41}{18}$$

$$x = \frac{31+41}{18} = \frac{72}{18} = \boxed{4}$$

$$x = \frac{31-41}{18} = \frac{-10}{18}$$

Not an integer

Cons. integers are 4 & 5.

3

Eric can do a job alone in 3hrs while Mark can do the same job in 5 hrs alone.

How long if they work together?

Work by Eric + Work by Mark = one Complete work

$$\frac{1}{3} \cdot t + \frac{1}{5} \cdot t = 1$$

LCD = 15

$$5t + 3t = 15$$
$$8t = 15$$
$$t = \frac{15}{8} = 1.875 \text{ hrs}$$

It takes Ashley 3 times longer than Taylor to do a job alone.

Together, they can do it in 21 days.

How long if they work alone?

Taylor → x
Ashley → $3x$

Work by Taylor + Work by Ashley = one Comp. work

$$21 + 7 = x$$
$$x = 28$$

Taylor 28 Days
ε
Ashley 84 Days

$$\frac{1}{x} \cdot 21 + \frac{1}{3x} \cdot 21 = 1$$
$$\frac{21}{x} + \frac{21 \ 7}{3x} = 1$$
$$\frac{21}{x} + \frac{7}{x} = 1$$

LCD = x

It takes Jerry 18 more minutes than Dan to wash a Car.
working together, They can wash the Car in 12 minutes.
How long if they work alone?

Dan → x
Jerry → $x+18$

$\dfrac{1}{x} \cdot 12 + \dfrac{1}{x+18} \cdot 12 = 1$

$\boxed{\dfrac{12}{x}} + \boxed{\dfrac{12}{x+18}} = \boxed{1}$

LCD = $x(x+18)$

$12(x+18) + 12x = x(x+18)$

$12x + 216 + 12x = x^2 + 18x$

$24x + 216 = x^2 + 18x$

$x^2 + 18x - 24x - 216 = 0$

$x^2 \quad - 6x \quad -216 = 0$

$a = 1 \quad b = -6 \quad C = -216$

$b^2 - 4ac = (-6)^2 - 4(1)(-216) = 900$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{6 \pm \sqrt{900}}{2} = \dfrac{6 \pm 30}{2}$

$x = \dfrac{6 + 30}{2}$

$x = 18$

$x = \dfrac{6 - 30}{2}$

$\cancel{x = -12}$

Jerry 18 mins
&
Dan 36 mins.

Simplify : $\dfrac{5}{x^2 - 2x - 15} - \dfrac{3}{x^2 - 25}$

$= \dfrac{5(x+5)}{(x-5)(x+3)(x+5)} - \dfrac{3(x+3)}{(x+5)(x-5)(x+3)}$

$= \dfrac{5x + 25 - 3x - 9}{(x-5)(x+3)(x+5)} = \boxed{\dfrac{2x + 16}{(x-5)(x+3)(x+5)}}$

Simplify: $\dfrac{3x^2 + 5x - 2}{9x^2 - 1} \div \dfrac{x^2 + 5x + 6}{9x^2 + 6x + 1}$

$= \dfrac{(3x-1)(x+2)}{(3x+1)(3x-1)} \cdot \dfrac{(3x+1)(3x+1)}{(x+3)(x+2)}$

$= \boxed{\dfrac{3x+1}{x+3}}$

Solve

$$\frac{x+3}{x^2-5x} \bowtie \frac{24}{x^2-25}$$

Hint: Cross-Multiply

$(x+3)(x^2-25) = 24(x^2-5x)$

FOIL   Dist.

$x^3 - 25x + 3x^2 - 75 = 24x^2 - 120x$

$x^3 + 3x^2 - 25x - 75 - 24x^2 + 120x = 0$

$x^3 - 21x^2 + 95x - 75 = 0$

Divide the LHS by $x-5$, doing long Division.

$x \square = x^3$

$x \square = -16x^2$

$x \square = 15x$

$$\begin{array}{r} x^2 - 16x + 15 \\ x-5 \overline{\smash{\big)}\, x^3 - 21x^2 + 95x - 75} \\ -\underline{(x^3 - 5x^2 \qquad\quad)} \\ -16x^2 + 95x - 75 \\ -\underline{(-16x^2 + 80x \qquad)} \\ 15x - 75 \\ -\underline{(15x - 75)} \\ 0 \end{array}$$

$x^3 - 21x^2 + 95x - 75 = 0$

Since we divided by $x-5$, and the remainder was zero, $x-5$ is a factor

$(x-5)(x^2 - 16x + 15) = 0$

$(x-5)(x-15)(x-1) = 0$

$\frac{28}{4} = 7$

$4 \cdot 7 = 28$

by Z.F.P.

$x - 5 = 0$ $x - 15 = 0$ $x - 1 = 0$

$x = 5$ $\boxed{x = 15}$ $\boxed{x = 1}$ $\{1, 15\}$

is an E.V.

The sum of two numbers is 5.
twice one of them reduced by 3 times
the other one is -5.
Use system of linear eqns to find
both numbers.

$3\begin{cases} x + y = 5 \\ 2x - 3y = -5 \end{cases}$

$$5x \qquad = 10$$

$\boxed{x = 2}$

$2 + y = 5 \qquad \boxed{y = 3}$

The numbers are 2 & 3.

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find eqn of a line that contains
$(5, 0)$ and $(0, -4)$.

$m = \dfrac{0 - (-4)}{5 - 0} = \dfrac{4}{5}$

$y - y_1 = m(x - x_1)$

$y - 0 = \dfrac{4}{5}(x - 5)$

$y = \dfrac{4}{5}x - \dfrac{4}{5} \cdot 5$
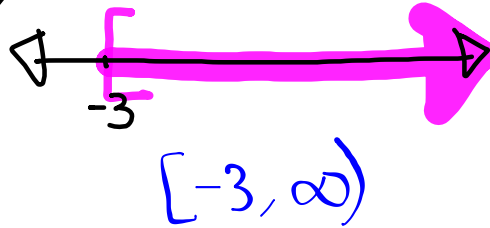
$\boxed{y = \dfrac{4}{5}x - 4}$

Solve & graph

$6 - 3x \le x + 18$

$-3x - x \le 18 - 6$

$-4x \le 12$

$\dfrac{-4}{-4}x \ge \dfrac{12}{-4}$

$x \ge -3$

-3

$[-3, \infty)$

Simplify $\left(\dfrac{-3x^{-4}}{2y^{-6}}\right)^{-4}$

$= \left(\dfrac{-3y^6}{2x^4}\right)^{-4} = \left(\dfrac{2x^4}{-3y^6}\right)^4 = \boxed{\dfrac{16x^{16}}{81y^{24}}}$

Divide: $\dfrac{25x^3 - 10x^2 + 5x}{-5x^2}$

$= \dfrac{25x^3}{-5x^2} - \dfrac{10x^2}{-5x^2} + \dfrac{5x}{-5x^2}$

$= \boxed{-5x + 2 - \dfrac{1}{x}}$

Class Qz

FOIL, Simplify, then Solve by using Quadratic formula:

$(3x + 1)(2x - 1) = 4$